Determining Characteristic Impedance of Transmission Lines with the MSA

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This document demonstrates methods of using the MSA to determine the characteristic impedance of coax cable or other transmission lines.

Review of Transmission Lines

Transmission lines commonly take the form of coax cable, twisted pairs, or parallel conductors. They have a characteristic impedance that determines the voltage-current relationship of the signal propagating down the line. For example, in a 50-ohm coax cable, the propagating signal voltage is V volts, then the current is V/50 amps.

The characteristic impedance is often just called impedance. Technically, however, the impedance of a transmission line, as for any component, is the extent to which it impedes the signal transfer. The impedance of a transmission line is hopefully near zero. In a transmission line with loss, the longer the transmission line, the greater the loss and the greater the impedance. The characteristic impedance, however, is independent of length, and is a characteristic of that particular type of line.

Characteristic impedance is often referred to as Z_0, but it is typically assumed to be a pure resistance and can also be referred to as R_0. Short transmission lines are typically assumed to be lossless. If a short line has significant loss, then its Z_0 will contain a significant reactive component. So lossy transmission lines have more complications than just the loss they introduce. We will see below that Z_0 of a normal coax can have a very significant reactive component below 1 MHz, and even as high as 10 MHz.

The major significance of Z_0 is that a transmission line terminated in its Z_0 will produce no reflections at the termination. Another way to state this is that a 50-ohm line terminated with 50 ohms will present an impedance of 50 ohms at its input. Any other terminating impedance may be "transformed" by the transmission line. Thus, the input impedance of a 50-ohm line terminated with 25 ohms will generally be neither 25 nor 50 ohms; its value will depend on the length of the line. A lossless line that is ¼ wavelength long makes an especially nice transformer, causing a terminating resistance of R to look like 2500/R ohms. That is,

$$Z_{in} = Z_0^2 / Z_L \qquad \text{(Equation 1)}$$

Z_{in} is the input impedance, and Z_L is the terminating impedance. A useful way to look at this is that if the terminating impedance is Z_0/N, the input impedance will be Z_0*N. If you start out at half of Z_0, you end up at double Z_0. Small impedances look big, and big impedances look small.

This transformer effect is most commonly used by leaving the transmission line unterminated (i.e. infinite impedance) or shorting it. The quarter-wavelength line causes the open to look like a short, and a short to look like an open. This is the same effect as just described, using N=infinity.

A separate document discusses in painful detail the mathematics involved in transmission line calculations, including the use of characteristic impedance with an imaginary component.

Measurement of Z_0 By Manual Calculation
The MSA can be used to evaluate certain characteristics of transmission lines. First, we can use the transformer effect of the quarter-wavelength line to determine Z_0. We first rearrange the transformer equation as follows:

$$Z_0 = \sqrt{Z_L Z_{in}} \qquad \text{(Equation 2)}$$

In Figure 1, we used the MSA in Reflection Mode to scan a nominally 50-ohm coax cable terminated with a 25 ohm precision resistor. ("Nominally", because most coax can have a characteristic impedance 1-2 ohms above or below the stated value.) The MSA actually scans S_{11}, but we chose to graph S_{11} Phase and Impedance magnitude, because the impedance shows a dramatic change, whereas S_{11} magnitude stays near 1.
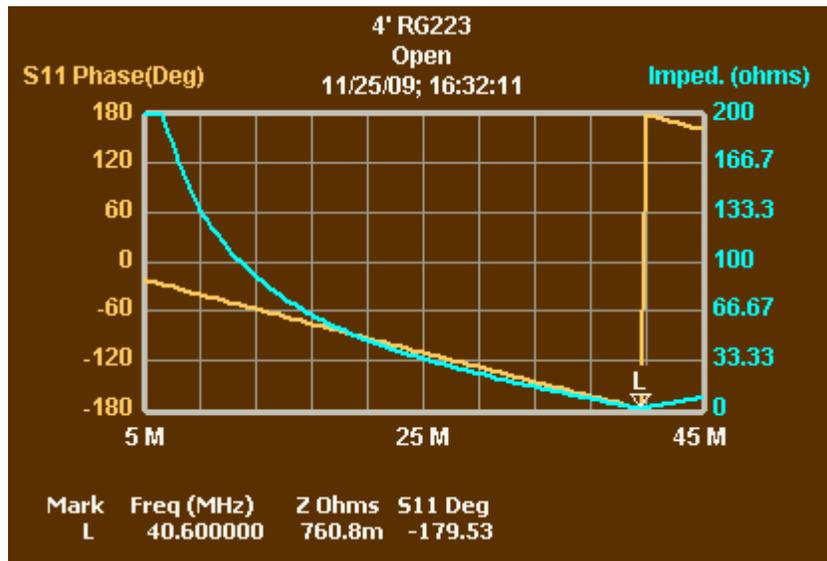
Figure 1—Coax terminated with 25 ohms
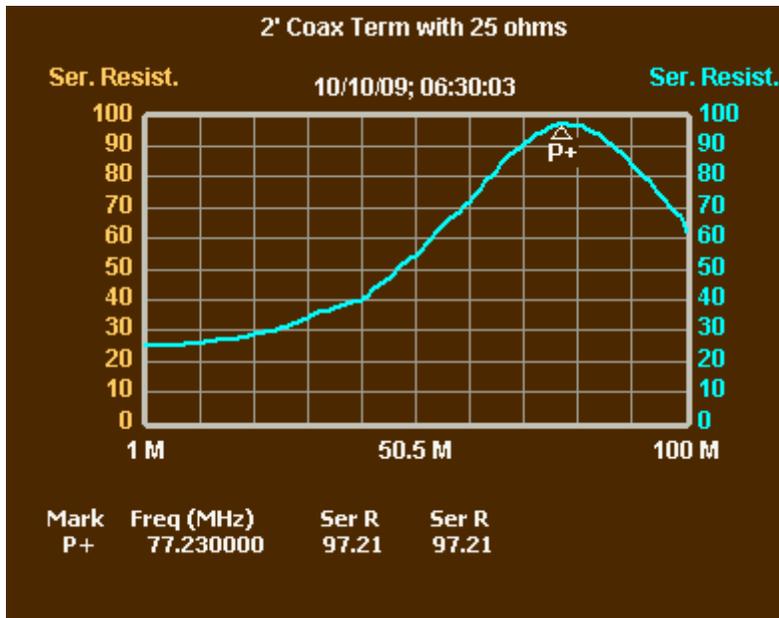Marker L is the quarter-wavelength frequency

The impedance is high at low frequency, and decreases to a very low value at Marker L, which is the quarter-wavelength resonance. At that point, S_{11} phase is very near -180 degrees, representing a pure resistance. Figure 2 shows the same coax, now terminated with a 60-ohm resistor.
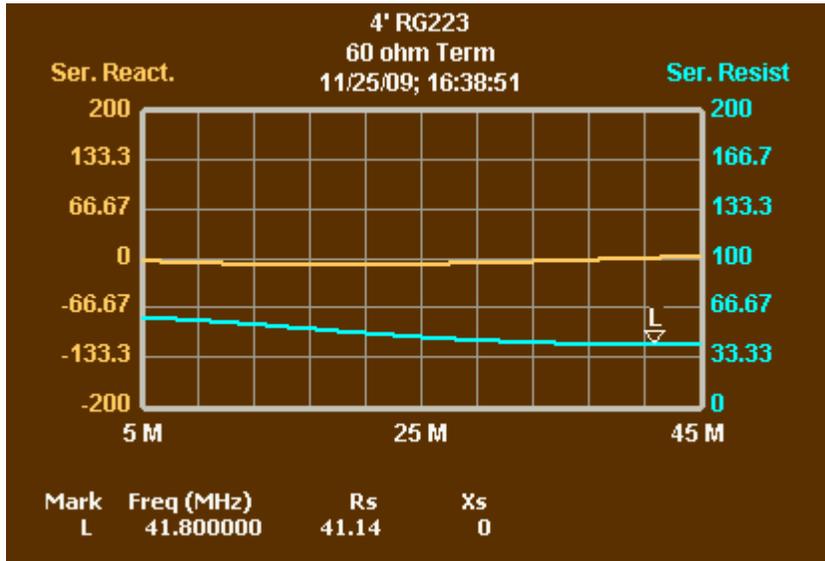
Figure 2—Coax terminated with 60 ohms
Marker L is the quarter-wavelength frequency

Figure 2 still shows a dip in resonance, but less pronounced. We now graph series reactance and series resistance. Marker L is at zero reactance, a slightly higher frequency than in Figure 1. This is probably due to the shallowness of the reactance graph, so a small error in reactance causes a significant change in the zero-crossing point. But this will not significantly affect our calculations. The resistance at marker L is 41.1 ohms. Plugging that value into Equation 2 as the coax input impedance, and 60 ohms as the termination impedance, we calculate Z_0=49.7 ohms.

Consider what would have happened had we terminated the coax with 25 ohms, which is smaller than Z_0. As noted above, at the quarter-wavelength frequency, big terminating impedances look small and small ones look big. The result is a peak, as shown in Figure 3. (This is a different coax of the same type.)
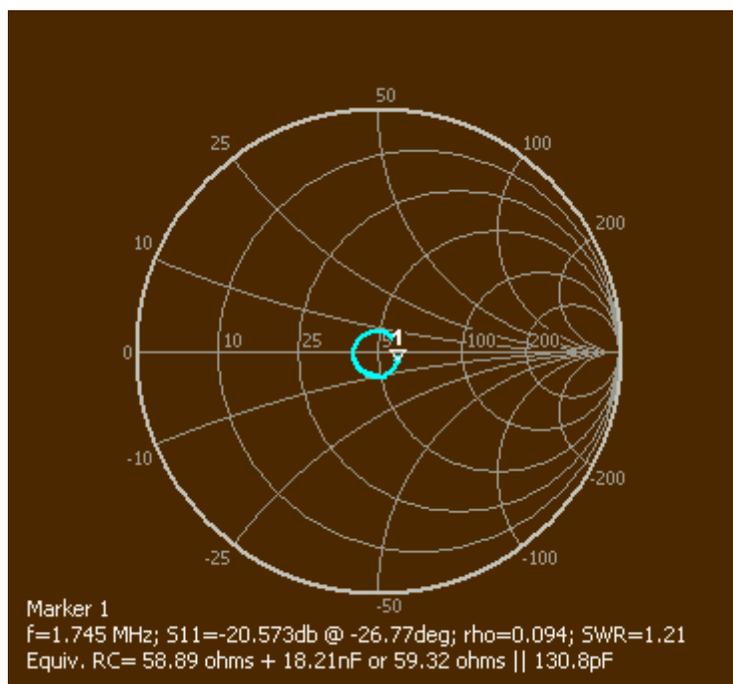

Figure 3—Coax terminated with 25 ohms
The peak is the quarter-wavelength frequency

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Here we have graphed just series resistance. The peak is sharp enough that it is easy to locate the top, without looking at phase. At the peak marked by the P+ marker, the 25-ohm resistor has been transformed into an input impedance of 97.2 ohms. Using Equation 2, we can calculate Z_0=49.3 ohms. This differs from the value of 49.7 ohms we calculated with the 60-ohm termination, but not by much.

The calculation is most accurate with a termination which is closer to the actual value of Z_0. A termination near 60 ohms would be an excellent choice for coax cables of unknown Z_0; 100 ohms would be perfect for twisted pairs.

Even without any calculation, this method can provide a ballpark estimate of Z_0, just by looking at the Smith chart. In fact, a termination of 60 ohms can give you a good idea of the characteristic impedance just by looking at the Smith chart, as shown in Figure 4.
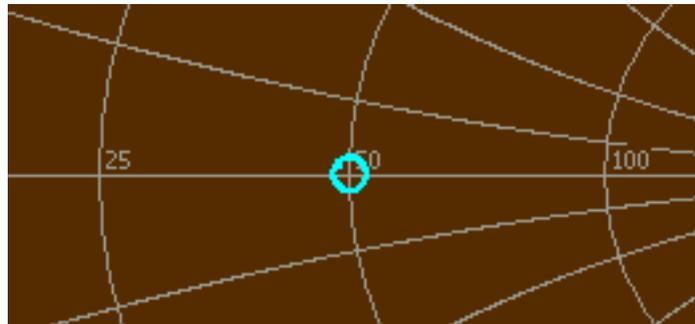

Figure 4—Coax terminated with 60 ohms
Scanned 1 MHz to 150 MHz

Figure 4 is another scan of 50-ohm coax terminated with 60 ohms. Note that the Smith chart draws a nice circle around the characteristic impedance. This does not tell you the precise value, but provides a nice approximation.

The circle in Figure 4 has a fairly large radius, reducing our accuracy in finding its center. We can reduce the circle radius by using a longer length of cable, with more total loss. To understand that concept, consider the Infinite Length Method of finding Z_0. If you measured the impedance of an infinite length of coax, the result would be the characteristic impedance, no matter what the termination of the coax. But you don't actually have to go all the way to infinity. If the cable is long enough that its losses substantially attenuate any reflection from the termination, the value of the termination will have negligible impact on the measured input impedance. If you have a significant length (more than 25 feet for modestly lossy cable, more than 100 feet for low loss cable), you can terminate it with a resistor approximating the characteristic impedance and measure at a high frequency (say,

200 MHz). The measured resistance will be very close to the characteristic impedance. (At high frequencies the imaginary part is likely to be small, so we ignore it here.) If the measured value is not flat over frequency, look at the Smith chart, where there will be a circle around the characteristic impedance, as shown below (you may need to maximize the Smith chart window to get a good view):
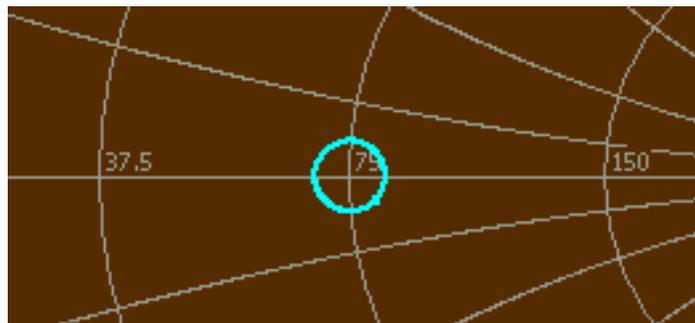

Figure 5—50' Coax terminated with 60 ohms
400-410 MHz

If the characteristic impedance is not 25, 50 or 100 ohms, it may not be clear exactly what point is circled. As an example, here is a 75-ohm cable terminated with 60 ohms.
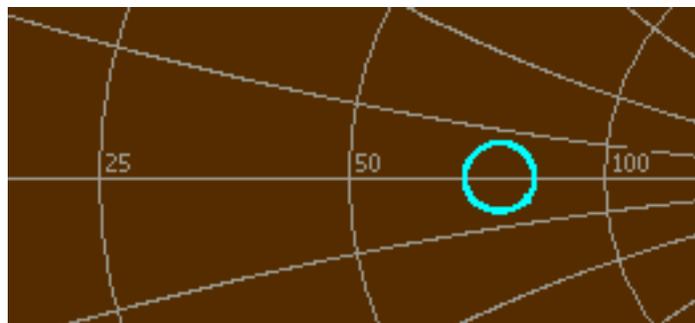
Figure 6—50' Coax terminated with 60 ohms
400-410 MHz

A nice circle is produced in Figure 6, but its center is not labeled. Make a guess as to its value, open the Sweep Parameters window, and enter that guess as the Graph R0. When you close the Sweep Parameters window, the Smith chart will be redrawn with that value at the center, and you will be able to verify whether your guess was correct. Here is the result of changing the Graph R0 to 75 ohms.

Figure 7—50' Coax terminated with 60 ohms; R0=75
400-410 MHz

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We now have the same circle, but it has moved to the center of the chart, which is now labeled 75 ohms. We now have a good estimate of the characteristic impedance.

The Coax Parameters Function

The above process is educational, but there is a way to simplify the process. In Reflection mode, the MSA has a Coax Parameters function (Functions➔Coax Parameters). You enter this function with a scan that includes the quarter-wavelength frequency. You specify the length or velocity factor of the coax cable and press Analyze; the software will calculate whichever one you did not calculate, plus the resonant frequency and the cable loss at that frequency (A_0); that loss is measured in dB per hundred feet. If you don't really care about precision in the velocity factor, an approximate value for length will be sufficient.

You may then calculate the Z_0 of the cable by pressing Analyze Z_0. The software will ask the value of the termination resistance. You can get decent results by specifying 10 M, and leaving the coax unterminated. But you get more accurate results attaching a termination that is within a factor of two of the actual Z_0, and entering that value. In the latter case, the software will perform a scan to measure the impedance with the termination in place. Z_0 is calculated by a method similar to that we used above, but the software will also take into account the full complex value of Z_0—the cable losses add an imaginary part.

Simulating Coax on the MSA

In the sweep parameters window, there is a Graph Data combobox which allows selection of the source of data to be graphed. Item 5, labeled Simulated RLC/Coax, allows simulation of a transmission line terminated by specified RLC components. By using a terminating impedance of zero or 1 G ohms, shorted and open transmission line stubs can be simulated for a specified length of a specified type of transmission line. There is a significant variation in parameters of coax of a given "type", based on manufacturer, so the assortment of coax cable types should be thought of as broad categories representing low, medium or high loss cables of various impedances. The regular graph of an open or short coax cable is pretty boring, because it just shows flat S_{11} very close to 0 and a sawtooth phase pattern. However, the Smith chart in Figure 8 reveals what is happening.
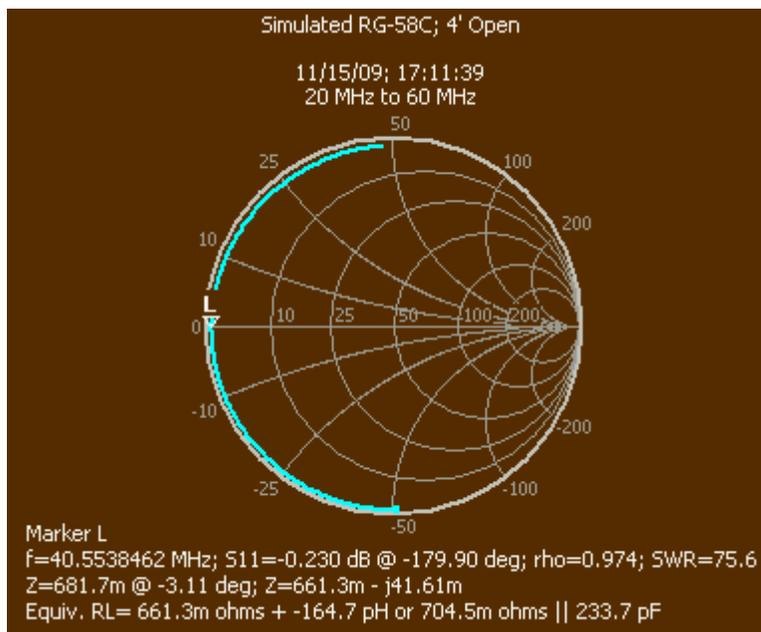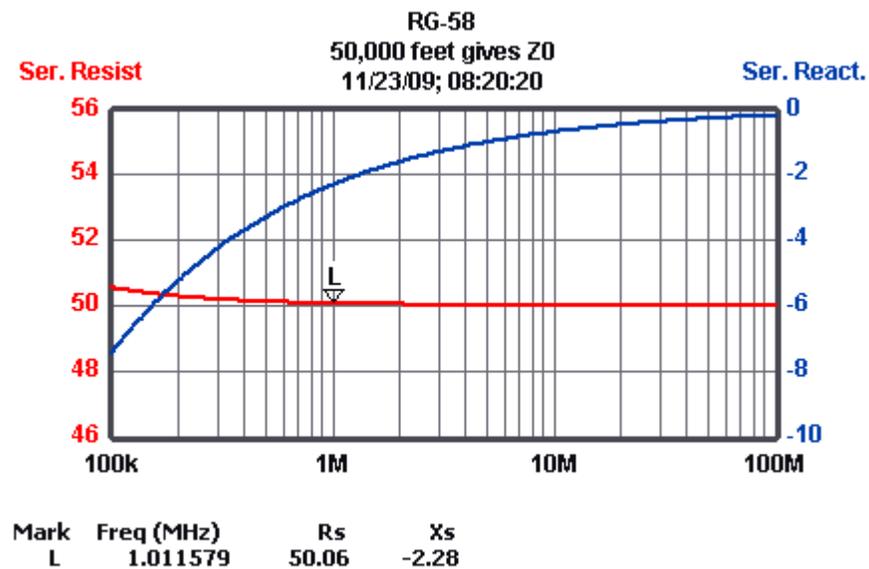

Figure 8—Smith Chart of simulated coax

The graph of Figure 12 starts near the bottom of the chart (the first point is slightly enlarged). The rho magnitude is near 1, and the angle steadily increases with frequency, causing a clockwise movement. The graph crosses the resistance axis at marker L, which is the quarter-wave resonance and exhibits very low impedance.

We can use such a simulation to determine the value of Z_0 for a particular type of cable, by creating a scan of the simulated cable and opening Functions→Coax Parameters, and proceeding as we did above. Because we are analyzing simulated data, several of the analyzed parameters will simply turn out to be what the simulator assumed them to be in the first place. However, in the case of Z_0, the simulator assumes the nominal value represents the real part of the characteristic impedance at higher frequencies where the imaginary part is small. Based on the coax losses, the simulator calculates Z_0 as a complex number at each frequency. If we specify an extremely long length (50 k feet) and a termination equal to the nominal Z_0, then negligible reflections will return to the beginning of the cable, and the measured input impedance of the cable will equal Z_0, with both resistance and reactance.

We are not likely to make use of the reactance in any manual calculations, but it is useful to see at what frequency the reactance becomes a small part of the characteristic impedance. Figure 9 shows a graph of Z0 for RG-58, based on simulated data and 50,000 feet of cable.

Figure 9—Smith Chart of simulated coax

The imaginary component (reactance) of the characteristic impedance is very significant at 100 kHz. It never goes away completely, but above 20 MHz is less than half an ohm.

Conclusion
We have shown that it is very straightforward to use the MSA to determine the characteristic impedance of a transmission line, and that consistent results can be obtained

if the line is of reasonable quality. Alternatively, simulation can be used to evaluate the imaginary component of a specific type of coax cable.