Hams and the Conjugate Match

July 16, 2001, by R. Dean Straw, N6BV
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In Chapter 4, "A View into the Conjugate Mirror" in the 2nd Edition of *Reflections* by M. Walter Maxwell, W2DU, (Sacramento: WORLDRADIO Books, 2001) on page 4-1, while discussing the concept of forward power measured by a directional wattmeter, Maxwell says:

> "The basis for understanding this rather subtle concept lies in the wave mechanics behind the principles of impedance matching introduced in Chapter 1 and defined in Chapter 2. As far as I know, the *wave* aspect of this subject has been presented in the literature only by Slater, Alford, (*Refs 35, 39,* and by this author in Chapter 23.) Perhaps this restricted exposure may account for some of the confusion in this area among engineers and amateurs alike."

Actually Looking at the Cited Reference

OK, let's actually look closely at the historically important *Ref 35* cited by Walter Maxwell. From J. C. Slater, in *Microwave Transmission*, (NY: McGraw-Hill Book Co., 1942), p 47 to p 48, when discussing matching theory, starting at equation (5.20):

"
$$\overline{Z_T} = R_T - jX_T = Z_{bb} - \frac{Z_{ab}^{\ 2}}{Z_{aa} + Z_1} \qquad (5.20)$$

> By a method entirely analogous to that used in deriving Eq. (4.65), we can show that the expression on the right of (5.20) is the impedance looking to the left from the arbitrary point where we are applying our condition, provided we replace the generator by an impedance equal to its internal impedance. Our result then, is that for maximum power transfer, the impedance looking to the right from our arbitrary point must be the complex conjugate of the impedance looking to the left from the same point; the reactances of the two half networks must be equal and opposite, an inductive reactance balancing a capacitive reactance, and the resistances must be equal. Obviously, our earlier theorem of (5.13) and (5.15), relating to a simple generator and load, is a special case of this more general theorem.
>
> The theorem we have just stated gives the general condition for impedance matching. If however the transmission line joining generator and receiver is resistanceless, so that there are no losses in it, we can prove a remarkable further theorem, no matter how complicated the line may be: if the conditions of the impedance match are satisfied at one point of such a lossless line, they are automatically satisfied at all points."

At this point, let me interrupt quoting Slater directly to point out that his "remarkable further theorem" above is what later authors, including Walter Maxwell, refer to as the "Conjugate-Match

Theorem." Later, on p 49, Slater says, after mathematically proving the "remarkable further theorem" in a series of equations (5.21) through (5.26):

> "We have now shown that if the impedance match conditions are satisfied at one point of a uniform resistanceless line, they are satisfied at another arbitrary point of the same line. Thus they are satisfied at another arbitrary point of that uniform line. This however is the beginning of another section of uniform line; if the conditions are satisfied at the beginning of this section, by our same theorem they are satisfied at any arbitrary point of it, and by extension of this method they are satisfied at any point of the composite line, so long as each section of the line is resistanceless, so that by (5.22) the coefficients of impedance are pure imaginary."

So Far So Good!

So far, so good for the conjugate-match theorem. All the right words show up in this venerable text. But we need to look at the *whole* story to completely understand Slater. He continues, in his next paragraph (still on p 49):

> "If there were losses in any section of the line, the corresponding impedance coefficients would have both real and imaginary parts, so that we could not perform the transformation from (5.25) to (5.24) by taking conjugates, and the theorem would not be true."

Any practical application in the real world involves lossy components, either in the transmission line or in any physical matching network. So, unless I've completely misunderstood Slater, it seems that for any practical application of the conjugate-match theorem, the conjugate-match theorem is *not applicable*.

In fact, logic dictates that a theorem that cannot be proved under specific circumstances (here, in the case of losses) cannot be called a "theorem" for this set of circumstances. Not to put too fine a point on it — the conjugate-match theorem is strictly theoretical, dealing only with lossless situations.

Is Slater the Only One?

And then let me ask a second question: Is Slater, whom Maxwell credits as being one of the pioneers developing the wave concept of the conjugate match, the only authority who states that the conjugate-match theorem doesn't actually come into play when losses are involved?

No, Slater isn't alone. One of the most widely recognized authorities on transmission lines is Walter C. Johnson, author of *Transmission Lines and Networks* (New York: McGraw-Hill Book Co., 1950). His book was cited by Walter Maxwell as *Ref 18*, and it states on page 191, in Section 7.7, Impedance Matching:

> "Therefore, maximum power in the load is obtained when the load impedance is the complex conjugate of the generator impedance. This condition is sometimes referred to as a "conjugate match."

Suppose that a generator is connected to a load through a lossless transmission line and a lossless matching device. None of the power is lost in the transmission system, and so, if the output of the generator is made a maximum by a conjugate match at its terminals, the power flow at all parts of the system must be a maximum. Then, if the system is opened at any point, the impedances looking in opposite directions must be the conjugates of each other. This can be made the basis for computing the matching elements to produce maximum power transfer."

Johnson in essence postulates the existence of a conjugate match everywhere, once a conjugate match has been established at the input of the line, by his stated condition that no power is lost and by stating that the transmission line is lossless.

Johnson doesn't explicitly state exactly what happens when components are actually lossy, the flip side of the conjugate match theorem. But then again, Johnson also does not dwell on the concept of conjugate matching elsewhere in his 354-page book. In the book's index, the term is shown only for page 191, the quotation shown above.

Any Clues Elsewhere?

So, let's go back to Slater to see if he gives us any clues about how we might look at things when we get to the real world — where real losses actually exist, both in transmission lines and in the matching networks used with transmission lines. Slater continues his analysis with a very interesting observation on p 50, middle paragraph (after he again had returned to the lossless line analysis), but as he now begins to looks at the situation from a different angle:

"The lossless transmission line connecting them has the properties of a transformer. We may replace the line by a four-terminal network."

Aha! Here's the approach that ultimately makes more sense in the real world, the world we hams live in. Rather than trying to force things everything into the purely theoretical construct of a conjugate match, let's look at the transmission line as though it were a *transformer*.

And please note the following, because it is very important. This transformer can be either a *lossless* or a *lossy* transformer. We have a way to handle either variety, easily.

A Different Way of Looking at the Situation

The Black Box Concept

In a way, Slater advocates a kind of *black box* approach. Put a load impedance on the end of this black box (aka, transformer or transmission line) and you'll get some other impedance at the input, depending on the black box's exact transforming qualities.

And just to be clear about it, this is basically the approach I took when rewriting Chapter 19 ("Transmission Lines") of *The ARRL Handbook* (editions since 1995), and both Chapter 24 ("Transmission Lines") and Chapter 25 ("Coupling the Transmitter to the Line") in the 18th and 19th Editions of *The ARRL Antenna Book*.

The hyperbolic transmission-line impedance equation analyzes a transmission line just as though it were a transforming device. Here's what it looks like:

$$Z_{in} = Z_0 \frac{Z_L \cosh \gamma\ell + Z_0 \sinh \gamma\ell}{Z_0 \cosh \gamma\ell + Z_L \sinh \gamma\ell} \qquad \text{(Eq 1)}$$

where

Z_L = complex load impedance at load = $R_L \pm j\, X_L$

Z_{in} = complex input impedance at input of line

Z_0 = complex characteristic impedance of line = $R_0 - jX_0$

γ = complex attenuation constant = $\alpha + j\beta$

α = matched-line attenuation constant, in nepers per unit length. Where α is expressed in dB/100 feet – then α is 0.1151 α/100 in nepers/foot.

β = phase constant, in radians per unit length. Expressed in terms of wavelength and the Velocity Factor (VF) of the line, this becomes:

$$\lambda = \text{VF}\frac{983.5691272}{F_{MHz}}\text{, in feet}$$

$$\beta = \frac{2\pi}{\lambda}\text{, in radians/foot}$$

Going Through All the Steps

And yes, Eq 1 above is rather intimidating! It's difficult to get a "feel" for it without actually making computations using it. That is the reason I have been writing software that does all the nasty complex-variable computations in Eq 1: *TL* (Transmission Line), *TLA* (Transmission Line Advanced) or *TLW* (Transmission Line for Windows). The latter two programs are available from ARRL, bundled with *The ARRL Antenna Book*. The *TL* program can even be downloaded free of charge from: http://www.arrl.org/notes/1867/index.html - software. It's part of the software supplied for *The ARRL Handbook*. All these programs can also design antenna tuner networks.

The logical steps to analyze an antenna, transmission line and matching network down at the end of the transmission line in the radio shack are thus:

1. Determine the impedance at the feed point of the antenna at a particular operating frequency — this is the load at the output of the transmission line. The feed-point impedance can be determined using a computer model, a theoretical mathematical model or by direct measurement.
2. Specify the type of transmission line (this sets the complex Characteristic Impedance, the Velocity Factor and the Matched-Line Loss for the line).
3. Specify the physical length of the transmission line.
4. Now, compute the impedance at the input of the transmission line, using the hyperbolic transmission-line equation Eq 1 (probably using a computer program).
5. Design a matching network to transform the impedance at the input of the transmission line to the impedance required by the transmitter (usually 50 Ω).

Actually, if the object of the exercise is simply to design a matching network, such as an antenna tuner, you could bypass steps 1. through 4. and simply *measure* the impedance at the input end of the transmission line down in the shack. That input impedance is what the antenna tuner sees at its output terminals.

What About a Lossless Line?

The hyperbolic transmission-line equation can easily handle a theoretically perfectly lossless line — simply set the matched-line attenuation constant α to zero. Simple as that. We have in Eq 1 a universal tool that ties together the theoretical and the practical.

How About a Really Big Black Box?

In fact, we could place our antenna, its environment and its feed line inside a *really large black box*. On the outside, this box would have two terminals, connected to the end of the feed line inside. Now, if you were to connect an impedance meter to these terminals you would measure a single, unique impedance at any given frequency.

For such a large black box, the concept of "conjugate match" has even less relevance — simply because the impedance measured at the terminals of this black box is totally determined by what's on the *inside* the box, not by anything external to the box (such as a transmitter's pi-network or an antenna tuner). We've done the ultimate in separating the antenna and feed line from a transmitter or an antenna tuner.

Doing the Analysis Yourself

Actually working through steps 1. through 4. is valuable, however, because it can give you a much better idea about what you should reasonably expect in a system. That's precisely why I included in Chapter 25 of *The ARRL Antenna Book* tables for typical multiband wire antennas. The impedances at the feed point for a 100-foot center-fed dipole 50 feet in height are listed in Table 1, and those for a 66-foot long Inverted-V dipole, 50 feet high at its apex are listed in Table 2 in Chapter 25.

These tables also show the computed impedances at the shack-end of a 100-foot long piece of nominal 450-Ω "window" ladder line. You can use the free *TL* program mentioned above to do these computations yourself, starting from the feed-point impedance of each dipole.

Again, the impedance at the input of the transmission line (or the terminals of our large black box above) is what the antenna tuner has to transform into 50 Ω for the transmitter. And this antenna tuner, with its hopefully small, but nonetheless inevitable, real-world losses cannot create a conjugate match throughout the entire system, because of the losses.

One Last Consideration About the Conjugate Match

Consider the situation I posed in the July 2001 issue of *QST* in the "QST Workbench, The Doctor is IN" column, on p 64. (Yes, I've blown my cover—I'm one of a number of people who contribute answers to this column.) Rather tongue-in-cheek, I posed a hypothetical situation.

Let's say I bought a very unusual transmitter at a hamfest somewhere. Instead of working into 50-Ω like all my friends' transmitters, mine is designed to work into a 120 –*j* 400 Ω impedance. Yes, this is a very weird impedance, but bear with me. Let's say that I want to operate on 40 meters one night, so I check my antenna system, just to be sure, with my trusty impedance meter.

At 7.1 MHz, I measure an impedance of 120 –*j* 400 Ω at the input of the transmission line going to my *Super-Duper Signal Scooper* all-band antenna. Boy, am I ever lucky — that's just what I need for my transmitter! So I directly connect the end of the transmission line to my transmitter, and when I fire it up, I get full rated power output, at the rated distortion level. Everything is just fine and dandy.

Now, did you find any hint of a conjugate match mentioned anywhere in this hypothetical scenario? Of course you don't, because there is very obviously no conjugate match involved here. There's no antenna tuner involved, and there's no pi-network in my unique, make-believe transmitter.

And unless you have a really unusual feed line with zero loss, and unless you have an antenna tuner with zero loss, you won't find a conjugate match in any other antenna system either because a conjugate match doesn't exist in anything but theory.

Let me be clear: I agree that the concept of conjugate match is useful for introducing the subject of wave reflections to a very technical audience. I do not consider it the best way to present the subject to hams. Both I and other ARRL technical staff consider the concept of representing a transmission line as a transformer much more relevant to hams than the conjugate match.

However, for those very technical hams, we have left references to Walt Maxwell's books and articles in *The ARRL Antenna Book*. That's only fair.

73,

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Some Background Information

I was trained at Yale University (1967, BS Engineering and Applied Science) as an electronics engineer. I worked for 25 years in industry, both as a bench engineer and as a manager, in engineering and technical marketing. I have been the editor of *The ARRL Antenna Book* since 1993, when I first joined HQ staff.

Although I consulted long and hard with all technical staff at HQ before doing so, I am the person who ultimately is responsible for removing "conjugate matches" from the ARRL books with which I have been associated; mainly *The ARRL Antenna Book* and *The ARRL Handbook*.

At the Dayton Hamvention in May, 2001, I had my first opportunity to meet Walt Maxwell, W2DU, in person. We discussed at some length the topic of conjugate matching. It was a useful and wide-ranging discussion, and I told Walt that we at ARRL took no "sides" on the personal controversy between him and Warren Bruene.

The fact that ARRL has published articles and letters by both men should testify that we are treating each of them fairly. Readers may have seen the on-going series of articles and counter-articles by both Walt and Warren published in *QEX* over the last year or so.

We are now, however, hearing from *QEX* readers that they are getting fed up with this never-ending controversy — the same kind of messages we received from *QST* readers when that magazine became the forum some years ago for this conjugate-match battle. "Enough already!" is the message that is coming through, loud and clear.

At Dayton, when Walt asked me directly why I had removed the extensive section on the conjugate match that he had written in earlier *ARRL Handbooks* I told him the following:

1) I am a RF engineer. I have designed and worked with HF and VHF power amplifiers ranging from 250 mW exciters to 250 kW military SSB transmitters.

2) Never, not once, did I consciously use the concept of the "conjugate match" to design and develop any of the above-mentioned transmitters.

3) The output matching networks used in any transmitter I have worked on have been designed using straightforward network equations. The goal was to transform an output load resistance, usually 50 Ω, with a specified level of SWR, to the plate, collector or drain load-line resistance needed to develop the specified output power, at a specified level of Intermodulation Distortion (IMD) and at a specified level of harmonic content. The transmitters also had to be reliable — meaning that the heat dissipation limits for all active and passive devices were kept within safe limits.

4) I know of no professional (or amateur) design engineer who has ever consciously used the concept of conjugate matching to design a transmitter — and, believe me, over the years I have asked many of them this direct question because of the continuing Maxwell/Bruene battle.

I have also talked to many people at conventions and many have said words to the effect: "I'm not a PhD level mathematician and can't argue the merits of either Maxwell's or Bruene's position. Heck, I can't even understand what they are arguing about."

Readers of *The ARRL Antenna Book* and *The ARRL Handbook* have expressed much the same sentiment over the years. They have, in essence, marveled at the complexity of Walt Maxwell's descriptions of the wave mechanics and the conjugate match, but at the same time many have not found the information to be of much practical value. I told Walt, face-to-face: Our readers have been forced to "take a drink from a fire hose."

As an editor, I have a responsibility to our readers to present information that is not only technically accurate but which is also useful and practical to them. I told Walt that it is my opinion that there are better ways to describe matching networks to Radio Amateurs — and that is the reason why I've removed the concept of conjugate matching.

It is not that the basic concept of the conjugate match itself is wrong — it is simply overwhelming and confusing to the average Radio Amateur, and most importantly, it only holds for lossless situations. Others may disagree with this judgment, and I should hope that we can agree to disagree, as gentlemen of good will.

And I agree wholeheartedly with what Walt Maxwell himself wrote in the first edition of his book *Reflections*, on page 19-7, in section 19.3:

> "Until now the discussion has concentrated only on obtaining delivery of the maximum available power from a source which we have considered to be a classical generator. Returning for a moment to the definition of the conjugate match given in Beatty's item 1 (Section 19.1), we see that one of the conditions required for maximum absorption of power in the load is that the load resistance equals the source resistance of the generator. However, in this respect we must be aware of a crucial difference between the classical generator and the output circuit of a real-world RF power amplifier. This difference has no effect on our routine matching operations, but it does affect the terminology we use if we are to have a technically accurate description of the matching operations.
>
> The difference is this: In the classical generator, the optimum load resistance required for delivering the maximum available power is always equal to the source resistance of the generator, as required for a conjugate match. However, because of the complex nature of various relationships between voltages and currents in an RF power amplifier, during normal operation the internal source resistance of the amplifier, RS, is almost never equal to its optimum load resistance, RL. Hence because of the different values of resistance existing between the source and the load when we complete a matching operation, we do not have a *true* conjugate match when an RF amplifier is coupled to an antenna tuner or a feed line. For the reader who has been conditioned to believe that the routine matching operation achieves a conjugate match, it may be helpful to understand that, in reality, the tuner or feed line is matched to the *optimum load resistance* of the amplifier, but it is conjugately *mismatched* to the source resistance."

So, in a nutshell, Walt Maxwell teaches that the concept of conjugate matching doesn't actually hold when it is extended to an RF power amplifier.

The concept of a conjugate match is useful when introducing the basic concepts of wave reflections (and Walt Maxwell does a thorough job in his book on this subject). But when you come down to practical applications of RF power amplifiers and real world, lossy transmission lines and matching networks, it is more useful to analyze things using precise mathematical models, such as transmission-line equations or straightforward network equations.

At Dayton, I also re-affirmed to Walt that the reason why ARRL decided not to reprint *Reflections* was a simple economic one. The demand had fallen off greatly towards the end of the years we sold the book. Simply put, the people who wanted to buy the book had already done so and reprinting it was not a good economic move.