APPENDIX 11 – ON THE FUNDAMENTALS OF IMPEDANCE MATCHING

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Considerations for Electrical Matching

The concept of matching is concerned with transferring energy from one entity to another. Energy, by definition, is the ability to do work. Of interest to this discussion is that energy is conserved. We rely on the principle that the amount of energy that is transferred cannot exceed the amount available. It can be less than the available amount, but not more.

In electrical networks we consider electrical energy that is converted to thermal energy to be *dissipated*. When calculating electrical efficiency such dissipated energy goes into the *loss* column.

To permit mathematical analysis of electrical networks a consistent pair of variables is defined; namely: voltage (an "across" variable), and current (a corresponding "through" variable). Energy level is defined as the product of these two variables, and the entire system of units is chosen and defined so that, within this model, the principle of *conservation of energy* is assured.

It is a mathematical fact that any specific value can be the product of an infinite number of pairs of multipliers. Thus, a specific energy transfer rate will not be completely defined without a statement of the value of at least two of the three variables involved — voltage, current, and power (the rate at which energy is transferred — the product of voltage and current); or a statement which includes any another combination of the variables and/or new variables from which at least two of the original variables can be uniquely determined. And, since the "law" of conservation is always to be true (at any instant of time) we must also account for any time-dependent variations of the variables.

Other applicable nomenclature includes the terms *source* (to label the entity from which energy is being transferred, and *load* (the entity to which the energy is being transferred). In the usual configuration a source and load are connected together so that energy transfers at some rate from the source to the load. All of the energy that is transferred passes through the connection.

Note here that we can expand on the idea of source to include entities within which transformation from other forms of energy takes place and those that only pass on electrical energy. For analyzing the rate at which either type will transfer energy to a load it is irrelevant which type of source is involved. We need only know about the characteristics of the available energy at the point of connection.

In one of the most common engineering models, at each connection in a serial network of entities passing energy we have both a source and a load. For purposes of analyzing the rate of energy transfer at a specified connection, we need to know sufficient information about the source and the load. Traditionally, in electrical engineering, for the source, we specify the maximum power level and the mathematical ratio of the voltage and current at which that power level is available. The ratio has been given the name *impedance*. For the load side of the connection, we specify the impedance at which the load will accept energy.

Of considerable interest, but of no relevance to the determination of the rate of energy transfer, is how the impedances of source and load are established. A simple resistor, for example, will present an impedance numerically equal to its electrical resistance. That is, a specific voltage will exist across it for a given value of current passing through it. Thus the instantaneous ratio of voltage to current which is defined by its electrical resistance is also identical to the ratio of voltage to current it presents as a load when connected to a source. In most cases, however, the situation is not nearly as simple. The load impedance presented by a transmission line is established by such factors as the impedance of the load connected at the other end (at which connection the line serves as a source), and perhaps also on the length of the line for situations where the variables are time variant.

Sources of the type where transformations from other forms of energy take place are particularly interesting. An electrical battery cell transforms chemical energy into electrical energy, which is then transferred to the battery's electrical load. The chemical processes within the cells and the number and the connection configuration of the cells determine the power level and impedance at which the energy is available from the battery. Note that the ratio of voltage to current at which the energy is available is not necessarily due to dissipative processes. In the case of common battery cells the chemical processes that determine the impedance are primarily non-dissipative.

Getting back to the analysis of energy transfer at the connections in a network, we should be able to identify some simple consequences of our model and some special cases where the results are somewhat intuitively obvious. The standard model tells us that a load that presents a less-than-infinite voltage/current ratio to the connection will respond to any applied voltage with a current flow. If the relationship between voltage and current is linear (ie: current and voltage are proportional to one another) and known (for both the source and the load), and we know one other value (such as the no-load voltage of the source) we have sufficient information to determine the rate of energy transfer across the connection. (Note that instead of the no-load source voltage we could use the source's short-circuited current, or we could use the power level established with any other known load connected.)

For any source with a proportional voltage/current relationship the product of voltage and current (power level) will have a maximum. (This is a characteristic of the mathematics and has nothing to do with such things as limits imposed by the components used in the network. Good engineering design assures that the practical limits exceed the mathematical limits.) Therefore, if a particular load results in a combination of voltage and current that differs from the specific values that result in the maximum power level, the energy transferred to the load will be less than the maximum available.

Consider a source with the simple relationship that the voltage varies linearly from 10 to 0 volts when the current varies linearly from 0 to 10 amperes. In other words, a characteristic of this source is that when current is drawn from its terminals the voltage across those terminals will decrease in proportion to the instantaneous value of the current. At either extreme the product of voltage and current is zero (i.e: 0 x 10 = 10 x 0 = 0). At 5 volts and 5 amperes we have a product of 25 (25 watts) which turns out to be the maximum value for all possible mathematical products of the voltage and current. All other combinations result in a product less than 25 watts. This is still the case even if the voltage and current vary in time. The instantaneous results must still be the same. The mathematics will just be more complicated. If the variation is sinusoidal the mathematics will again be relatively simple. In any event, the maximum rate of energy transfer from source to load occurs at one specific operating point for any system where the variables are proportionally related.

Another way to look at this principle is to consider that in order to take all possible energy from the source the load must simultaneously make use of all of the voltage *and* all of the current at the operating point where the source is at it's maximum power level.

For the single path, series connected network we have been discussing the source and load voltages and current levels will always be equal in magnitude at the connection. To keep the mathematics consistent we need to do something about the arithmetic sign for the "through" variable, current. The common convention is to define currents flowing into an entity as positive. Thus the load current will be positive when the source current is negative. The mathematics then tells us that when the voltage applied to a simple load is increased the current will also increase in proportion — the constant of proportionality being the load's voltage/current ratio (impedance). Using this same convention for the source we have a negative proportionality (the source's impedance) that says the voltage decreases with increasing current. Further convention is to differentiate the two by simply calling them the *source impedance* or *load impedance* without use of the sign. Note that a given device can exhibit a source impedance that is completely different from its impedance as a load. A primary chemical battery cell is just one example. A diode is another.

Now let's take a careful look at the condition where the load is such that the source is operating at the point where maximum energy transfer is taking place. The voltages and currents are the same magnitude for both source and load and the voltage-current product is at the maximum for the source. Since the current level is determined by the load as a function of the source voltage it must be that the characteristic voltage/current ratio for the load is the same as that for the source. Using the simple source described above as an example, if the load is such that an applied 5 volts results in a 5 ampere current then the power level in the load is 25 watts. Since that is also the maximum power level for that source it must be that any other load condition will result in a lower power level.

But, what if the source voltage and current are time varying and not in phase? Let's assume our simple source is operating as a sinusoidal source (AC) and the values expressed for the variables are RMS values. The mathematics still produces the same results. We can cause a phase difference between the source voltage and current by introducing a reactive element (say a capacitor in parallel with the output terminal). This will result in a different instantaneous voltage/current ratio for the source. (i.e: The source impedance is now different because of the introduced phase shift.) For the same load connected to the source the instantaneous product of voltage and current will be less than the original 25 watts because of the phase difference. If the phase difference can be reduced or eliminated the instantaneous product can be increased up to the maximum 25 watt level. This can be accomplished simply by placing an appropriate inductor across the load's terminals.

I find it interesting to note that if the two reactive elements (the capacitor attached to the source and the inductor attached to the load) are considered together they form a simple parallel LC circuit across the connection. Because the reactance of each is the same value this LC circuit is resonant at the operating frequency and thus takes no energy from the connection. It also draws no net current from the connection at any voltage so it effectively displays an infinite impedance to the connection. Thus we can eliminate it from consideration and we are right back to the simple case.

But what about the impedances for the modified source and load? They are not the same because of the differences caused by the two different reactive elements. The convention is to state the out-of-phase voltage/current ratio as a complex number with the imaginary part of the number representing the reactive part. Because the reactances for the source and load are equal in magnitude but opposite in direction their impedances are simply the conjugates of one another.

Thus we can conclude that to design a load to operate at the mathematical maximum power level when connected to a specific source, the impedance of the load must be the complex conjugate of the source impedance. Any other value results in a lower power level.

Conversely, if the maximum available power level (as determined by the math not the component ratings) is realized through adjustment of the load impedance then it follows that the source impedance must be the complex conjugate of that load impedance.

If the characteristics of a source are such that it behaves approximately in a linear fashion, the conditions for matching to obtain maximum energy transfer will also be approximately the same.

Determining the output characteristics of a non-simple device, such as an RF amplifier using a pulsed active component as a part of its circuitry, is not a simple process. The usual RF amplifier also includes parallel energy transfer paths that further complicate rigorous analysis. The non-linear action of the active device in class-B and class-C circuits is hidden from the output terminal through the combined effects of the parallel energy paths and the energy storage capabilities of the tank circuit.

It has been my experience that the output characteristics of even class-C amplifiers are very close to those of ideal linear sources, and I thus conclude that there is no valid reason to preclude use of the various theorems for analyzing energy transfer to loads connected to these devices. Determination of an amplifier's output impedance is probably best accomplished by finding the load that results in the maximum power level and then using the Complex Conjugate Theorem to define the impedance of the source. If the amplifier's output is reasonably linear it is not even necessary to find the point of maximum power. Simply determining two operating points may be sufficient to extrapolate to the impedance.

Suggestions that the theorems don't apply because of non-linear activity somewhere within the bowels of a device makes no logical sense. For purposes of analyzing energy transfer from one entity to another the only things that matter at all are the characteristics displayed at the terminals.