APPENDIX 9 – BASIC AXIOMS OF THE CONJUGATE MATCHING THEOREM

It is well known that when a load impedance differs from the source impedance, a matching device is required to permit delivery of all the available power from the source to the load. In this condition we say the load is 'matched' to the source. The term 'matched' has been used universally for many decades to describe this condition. In earlier days, the term was used alone. However, when all the power available from the source is delivered to the load, the matching occurs because the source and load impedances are conjugates of each other. During the last fifty years, the term '*conjugate* match' gradually came into use synonymously with 'match' to describe the term more accurately. In other words, 'match', as used in this context, and 'conjugate match' are often used interchangeably with no difference in meaning. Unfortunately, misinterpretation and misunderstanding of *conjugate* in the newer term has created confusion for many people when a routine impedance match is referred to as a 'conjugate' match. However, do not confuse 'impedance match' with 'conjugate match', because there can be an impedance match at one or more points in a system without necessarily having a conjugate match in the entire system. For example, an impedance match (but a non-conjugate match) can exist at the junction of a transmission line and its load, while a mismatch may exist between the source and the input of the transmission line.

Now for the basic axioms of the conjugate matching theorem:

Axiom 1) There is a conjugate match in an RF power transmission system when the source is delivering all of its available power to the load. *(Refs 120, 137, 35, 17, 69)*

Axiom 2) There is a conjugate match if the delivery of power decreases whenever the impedance of either the source or load is changed in either direction. This follows from the Maximum Power-transfer Theorem. *(Refs 17, 69)*

Axiom 3) If there is a conjugate match at any junction in the system, and if there are no active or 'pseudo active' sources within the network, there is a conjugate match **everywhere** in the system. (The phasors at any point along a transmission line are conjugates). *(Refs 35, 17, 69)*

Axiom 4) The term 'conjugate match' means that if in one direction from a junction the impedance is $R + jX$, then in the opposite direction the impedance will be $R - jX$. *(Refs 17, 69)*

Quoting Dr. W.L. Everitt's statement on the conjugate match: (*Refs 17, 69)*

"**Theorem** — If a group of four-terminal networks containing only pure reactances [*includes lossless transmission lines*] are arranged in tandem to connect a generator to a load, then if at any junction there is a conjugate match of impedances, there will be a conjugate match of impedances at every other junction in the system."… [*Parentheses mine*]

"If the dimensions [*values*] of the network elements are such that there is a conjugate match at any one of the junctions, there must be a conjugate match at all the junctions. Since the networks contain only pure reactances, there can be no dissipation, and all the power absorbed at the input of the first network must be transferred to the output. If at any junction there should not be a conjugate match, then by adjusting the impedance beyond the junction an increased absorption of power could be obtained. This would require an increase in power delivered by the generator, which is an impossibility. Therefore there must be a conjugate match at all junctions."…

"The Conjugate Theorem also shows that in a sequence of matching networks it is necessary to match at only one junction if the networks are non-dissipative. In actual practice, since there is usually some dissipation, it is frequently desirable to adjust at more than one point. An example is a radio transmitter feeding a line, which in turn is coupled to an antenna. If the line were non-dissipative it would be only necessary to adjust the matching conditions at one point."

Quoting from Robert W. Beatty, NIST, *Microwave Mismatch Analysis, (Ref 120)*:

1) Conjugate match—The condition for maximum power absorption by a load, in which the impedance seen looking toward the load at a point in a transmission line is the complex conjugate of that seen looking toward the source.

2) Conjugate mismatch—The condition in the situation above in which the load impedance is not the conjugate of the source impedance.

3) Conjugate mismatch loss—The loss resulting from a conjugate mismatch.

[Conjugate mismatch loss is not a dissipative loss, but is identical to reflection loss, which represents only the inability of the source to deliver all of its available power to a mismatched load.]

4) Z_O match—The condition in which the impedance seen looking into a transmission line is equal to the characteristic impedance of the line.

5) Z_O mismatch—The condition in which the impedance seen looking into a transmission line is not equal to the transmission-line characteristic impedance Z_O. In general, a conjugate match is a case of Z_O mismatch.

6) Z_O mismatch loss—The loss resulting from a Z_O mismatch, which is canceled by the reflection *gain* obtained with a conjugate match *(Ref 19, Sec 4.1.3)*.

7) Conjugate available power—*Maximum* available power.

8) Z_O available power—The power a source will deliver to a Z_O load.

More from Robert W. Beatty on the conjugate match in terms of 'conjugate *mismatch' (Ref 137)*:

"Conjugate mismatch is useful when one is given the available power P_A from a given generator, and wishes to determine the power P_1 that it will deliver to some speci-

fied load. If the powers are given in decibels relative to some convenient level, one subtracts the conjugate mismatch loss from the available power to obtain the power delivered to the load.

Definition: Conjugate mismatch loss is the ratio, expressed in decibels, of the available power from the generator to the power absorbed by the load connected to the generator."