New and Improved Formulas for the Design of Pi and Pi-L Networks

Published equations relating to the design of pi networks are often inaccurate where circuit operating Q is concerned. If Q is of primary consideration, use the equations in this article.

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This article introduces new formulas for the design of pi networks and pi-L networks. These new formulas permit the network to be designed based on the resistances to be matched and on the actual circuit operating Q value, Qo. Design formulas now in common use have only a partial circuit Q value as the circuit Qo. In many practical design cases the error involved in the usual formulas can be quite large and the resulting network will not perform as intended.

The pi networks, the pi-L networks and the resonant-L networks considered in this article are only those that are configured in the low-pass arrangement with the inductance as the series element and capacitors as the shunt elements, as shown in Fig. 1. This is the arrangement in which these circuits are invariably used in amateur rf applications. These networks, consisting of two, three or four reactance elements, have only one absolute requirement: At least one of the reactances must be of opposite sign to the others for a "match" or transformation of a load-end R2 to a desired R1 value at the source end. This is because resonance is impossible otherwise. A low-pass configuration is not required for a match, but it is the only arrangement of any importance in rf uses of such networks and therefore the only arrangement considered in this article.

The "Old Standard" Equations

The old standard pi-network design formulas are those familiar equations that have been published in the ARRL *Handbook* for many years, in ARRL *Solid State Design,* the *ARRL Electronics Data Book,* the *Radio Handbook,* and Motorola *Application Note AN-267.* They appear in Fig.

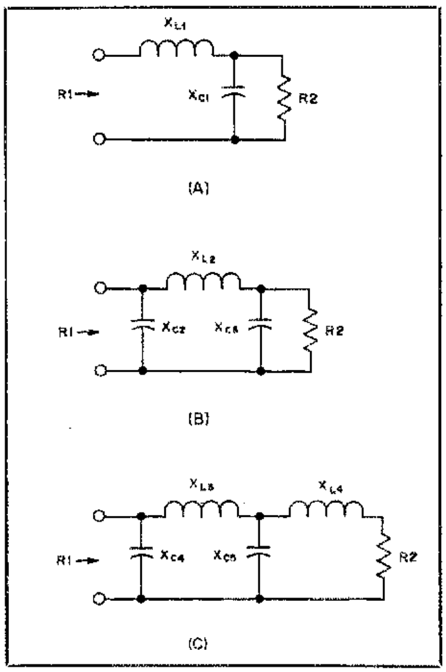

Fig. 1 — An L network is shown at A, a pi network is at B, and a pi-L network is at C. These networks are shown in the low-pass configuration. Each network matches the load, R2, to the input resistance, R1.

2, and apparently were first published in the amateur literature by Pappenfus, WØSYF, and Klippel, WØSQO, in 1950.[1] The tabulated pi and pi-L network data given in current editions of the ARRL *Handbook* are based on these formulas, as is similar data in the *Radio Handbook,* 21st edition.

These old standard formulas are derived by treating the pi network as two back-to-

$R1 > R2$

$$N > \sqrt{\frac{R1}{R2} - 1}$$

$$X_{C2} = \frac{R1}{N}$$

$$X_{C3} = \frac{R2}{\sqrt{\frac{R2}{R1}(1 + N^2) - 1}}$$

$$X_{L2} = R1\,\frac{N + \frac{R2}{X_{C3}}}{N^2 + 1}$$

Fig. 2 — The "old standard" equations for pi-network design. These equations are those appearing on page 54 of *The Radio Amateur's Handbook,* 55th (1978) edition, changed here for agreement with the reactance designations shown in Fig. 1. In these equations, N is the same as Q1 in the article.

back L networks. (The derivation is given in the appendix.) These formulas give correct results for the network reactance values for matching an R1 source end to the R2 load-end resistance. The only error involved is in assuming that the Q1 source-end L-network Q value used in the formulas is the resultant total pi-network Qo value. It is not, and understates the Qo value by the amount of the Q value of the load-end L network section, Q2.

In terms of the assumed formula Q1 value and the R1 and R2 to be matched,

$$Q2 = \sqrt{\frac{R2}{R1}(Q1^2 + 1) - 1} \quad \text{(Eq. 1)}$$

This is the error involved in taking Q1 as the Qo value, since Qo = Q1 + Q2. The error becomes progressively greater as the ratio R1 to R2 becomes lower. For R1/R2 = 1, the error is 100%, as Qo equals

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[1]Notes appear on page 29.

2 × Q1. The error is greater for R1 × R2, a common arrangement in matching the required R1 load resistance of rf transistor amplifiers to the usual 50-ohm R2 load.

If the R1-to-R2 ratio is moderately high, say 20 to 1 or higher, then the error in taking Q1 as the circuit Qo will be relatively small. Therefore the old standard formulas may be used for the design of pi-network output tuning circuits for the usual tube-type rf amplifiers. For example, for a pi-network design of R1 = 2500 ohms, R2 = 50 ohms and Q1 = 12, the actual Qo will be 13.38 instead of 12, and this error is unimportant. This design would be representative for a pi network for rf linear amplifiers operated at the 2-kW peak level at a plate voltage of 3000 V or so, as is common for current amateur-band linear amplifiers. About the worst case for an amateur tube-type design would be four sweep tubes in parallel requiring a plate-load resistance on the order of 400 ohms. Then, with a formula Q1 entry of 12 and a 50-ohm load, the actual network Qo would be 16.14 — too high, perhaps, but still workable.

The conclusion is that the old standard pi-network formulas provide satisfactory pi-network output tank coupling circuit designs for amateur tube-type rf amplifiers and have done so for more than 30 years. The Q1 value used for Q differs only moderately from the correct circuit Qo.

Unsatisfactory Q Results

The old standard formulas do not give satisfactory designs for Q-based pi-network circuits intended for the fixed-tuned input circuits of cathode driven (grounded-grid) rf amplifiers. The rf exciter normally requires a 50-ohm load, and the fixed-tuned pi-network circuit must match this R1 source end to the rf-amplifier cathode-driving impedance at the R2 load end. At a minimum this load is perhaps 50 ohms, and it may be as high as 100 to 150 ohms or so for some tube types and operating conditions. For the tubes currently popular for the amateur services — zero-bias triodes operated as grounded-grid Class B linear amplifiers — the cathode driving impedance will be in the range of 50 to 75 ohms when operated at the 2-kW-PEP level.

Since the input tuned circuit is fixed tuned to obtain a broadband response, the Q of the circuit is kept at the lowest level that will just provide an adequate "flywheel" effect. In practice, a Q of 2 to 3 is used. If the network is designed using the old standard formulas for the "best" case (where R1 = 50 ohms, R2 = 50 ohms and a Q of 3 is chosen), an actual circuit Qo of 6 will result. The error is 100%. The error will increase as R2 is larger than 50 ohms, as it usually is. Because of the higher than intended (or needed) Qo, the input circuit will not be sufficiently broadbanded, and solid-state broadband exciters may not be able to drive the amplifier to the full intended power level because of SWR turndown in the exciter. This might be the case especially on the relatively wide 75- and 80-meter band. These fixed-tuned-input pi networks should be designed using the formulas given later in this article.

As an example of the very large errors that can occur in the Qo value when pi networks having low R1/R2 ratios are computed by the old standard formulas, the first data entry in Motorola *AN-267* indicates a Q value of 1 for the network that matches an R1 of 1 ohm to the R2 load of 50 ohms.[2] The actual Qo for this network is 10.95, almost 11 times the stated Q value.

Pi-L Networks

The standard formulas are not suited for the design of pi-L networks in which the circuit Qo is an important factor, as it is in tube-type and other rf amplifiers. The reason for this is that in the pi-L the output load resistance, R2, is stepped up to an intermediate value, Rm. Rm becomes the load-end resistance value for the input-end pi-network section. Then, the ratio of R1 (source end) to Rm (load end) for the pi-network section is quite a bit lower than the R1-to-R2 ratio in a straight pi-network tank. The Qo of the pi- network input section of a pi-L will therefore have an added increment of Q caused by the lower ratio R1 to Rm. In addition, there is an added increment of Q from the output-end L network. The resulting error in assuming Q1 as the network Qo is large.

As an example of the large error involved, the pi-L tabulated data in Table 10 on page 6-31 of the ARRL *Handbook* (1979 and later editions) indicates a circuit Qo of 12 for the network data given. For that data, the true circuit Qo for the high end of the band varies from 19.47 for Z_{in} = 1500 ohms to 16.29 at Z_{in} = 8000 ohms. The errors are too large, and the networks given will have unnecessary additional losses because of the higher-than-optimum Qo of 12. In practice, the pi-L output tuning network would be expected to accommodate the line input variations for a line having an SWR of 2, at least. Tuning the network to match actual impedances that may be encountered could result in the pi-L network Qo value going well above 20. The old standard formulas should not be used for the design of the pi-network section of a pi-L tank network circuit.

Two sets of formulas and procedures are given later for the design of pi networks and pi-L networks. These procedures are based on the R1 and R2 to be matched and on the desired circuit operating Qo value. Both of the formula sets are simple algebraic formulas similar to the old standard formulas. They are equally simple and straightforward in use, but with the decided advantage of giving precisely correct results for all quantities.

It was noted earlier that the old standard Q-based pi-network formulas would provide satisfactory design for a pi-network output coupling circuit for most amateur tube-type rf amplifiers. Only a small-to-moderate error in the actual circuit Qo value will result. Since the new formulas given later are equally simple and straightforward, and since they give exact results for all quantities for any possible Q-based pi network, it is recommended that they be used for the design of all pi networks in which Q is a factor.

Gibson, G2BUP, published an article pointing out the Qo error in the old standard formulas and giving a graphical procedure for a correct pi-network solution.[3] This material was later reprinted in another publication.[4] In private correspondence in 1978, Gibson clearly demonstrated that the Q1 or input L-net-section Q value used in the pi-network formulas did not account for all of the stored energy or reactive power associated with the output capacitor, C2, or its equivalent capacitive reactance, X_{C2}. The actual circuit Q is given by

$$Qo = Q1 + Q2 \qquad \text{(Eq. 2)}$$

or

$$Qo = R1/X_{C1} + R2/X_{C2} = X_L/Rs \qquad \text{(Eq. 3)}$$

where

Rs is the series equivalent resistance value of the parallel-to-series conversion of the R2 and X_{C2} components.

The standard pi-network formula Q error was also noted by Whyman, W2HB, and by Kajii, JA1FG.[5,6] Kajii gave a graphical design procedure to obtain a pi-network solution based on the correct value of the circuit Qo. The graphical design procedures have the limited accuracy and the limited range of variables inherent in charts and graphs. The algebraic formula methods do not have these limitations.

The algebraic formula procedures given below as Procedure 1 and Procedure 2 are intended so the user can simply plug in the network R1, R2, and Qo values and crank out the network reactance values. Use of Procedure 1 or Procedure 2 is at the choice of the user. The formulas and procedures are simple to use, but close attention must be paid to circuit-element designation including subscripts and superscripts. This is particularly so in the pi-L-network solution, which has the added L network to account for.

To avoid the clutter involved in adding too much detail in the procedures, some preliminary background basic ac circuit material is shown in the appendix. This includes the series-to-parallel and parallel-to-series conversion formulas. An addition to these results provides a resonant circuit, including a resonant L network, which is the basis for deriving the old standard back-to-back L-net-derived pi-network formulas that are given. The fundamental energy-related Q is shown for the circuits for circuit components involved.

Of the two new procedures and formula sets given here, either will yield precisely correct design of pi networks and pi-L networks as used in amateur practice. This first set is based on an adaptation of the standard Q-based back-to-back L-network derived formulas. The second set is based on an adaptation of the non-Q-based general matching pi-network equations given by Everitt.[7]

Procedure 1, Pi Network

1) Refer to Fig. 3 and select the desired Qo value. Qo must be selected to satisfy the following:

$$Qo^2 > \frac{R1}{R2} - 1 \text{ and } Qo^2 > \frac{R2}{R1} - 1$$

2) Compute Q1.

$$Q1 = \frac{R1 \cdot Qo - \sqrt{R1 \cdot R2 \cdot Qo^2 - (R1 - R2)^2}}{R1 - R2} \quad \text{(Eq. 4)}$$

3) Then,

$$Q2 = Qo - Q1 \quad \text{(Eq. 5)}$$
$$X_{C1} = R1/Q1 \quad \text{(Eq. 6)}$$
$$X_{C2} = R2/Q2 \quad \text{(Eq. 7)}$$

$$X_L = \frac{R1 \cdot Qo}{Q1^2 + 1} \quad \text{(Eq. 8)}$$

4) For the special case where R1 = R2, select a value of Qo greater than zero. Then

$$X_{C1} = 2 \cdot R1/Qo \quad \text{(Eq. 9)}$$
$$X_{C2} = 2 \cdot R2/Qo \quad \text{(Eq. 10)}$$

$$X_L = \frac{R1 \cdot Qo}{\frac{Qo^2}{4} + 1} \quad \text{(Eq. 11)}$$

Note: Eqs. 9 through 11 are valid only for the special case where R1 = R2.

Procedure 2, Pi Network

1) Refer to Fig. 3 and select the desired Qo value to satisfy the following:

$$Qo^2 > \frac{R1}{R2} - 1 \text{ and } Qo^2 > \frac{R2}{R1} - 1$$

2) Compute X_L.

$$X_L = \frac{Qo(R1 + R2) + 2\sqrt{R1 \cdot R2(Qo^2 + 4) - (R1 + R2)^2}}{Qo^2 + 4} \quad \text{(Eq. 12)}$$

3) Compute Q1 and Q2.

$$Q1 = \sqrt{\frac{Qo \cdot R2}{X_L} - 1} \quad \text{(Eq. 13)}$$

$$Q2 = Qo - Q1, \text{ or}$$

$$Q2 = \sqrt{\frac{Qo \cdot R2}{X_L} - 1} \quad \text{(Eq. 14)}$$

4) Then,

$$X_{C1} = R1/Q1 \quad \text{(Eq. 15)}$$

$$X_{C2} = R2/Q2 \quad \text{(Eq. 16)}$$

Note: Eqs. 13 and 14 for determining Q1 and Q2 above are valid only for Procedure 2, using X_L from Eq. 12 and the variable values Qo, R1 and R2 used in Eq. 12.

The Pi-L Network

For the following discussion, refer to Fig. 4. Select the intermediate Rm value that is supplied by the output L net as the load for the input pi network. For example use $Rm = \sqrt{R1 \cdot R2}$ or other as appropriate within the requirement that R1 > Rm > R2. Then compute the L-net Q value.

$$Q_L = \sqrt{\frac{R_m}{R2} - 1} \quad \text{(Eq. 17)}$$

Then

$$X_{L2} = Q_L \cdot R2 \quad \text{(Eq. 18)}$$

$$X_p = Rm/Q_L \quad \text{(Eq. 19)}$$

Select the desired pi-L network Qo value. Next, compute the required pi-network-section Q value.

$$Qo_\pi = Qo - Q_L \quad \text{(Eq. 20)}$$

Use either the Procedure 1 or Procedure 2 pi-network formulas to compute the values for X_{C1}, X_{C2} and X_L for the pi network. Use the value specified for Rm as the value for R2 in the selected procedure. Also, use the Qo_π value for Qo. Note that the X_{C2} value obtained will be the X'_{C2} value in the pi-L network arrangement. Then,

$$X_{C2} = \frac{X'_{C2} \cdot X_p}{X'_{C2} + X_p} \quad \text{(Eq. 21)}$$

This equation yields the pi-L X_{C2} value. This completes the pi-L solution.

A program in BASIC for the Procedure 1 pi-network formulas is given in Table 1. Table 2 gives reactance values for a Qo value of 12 for pi-network tube-type tank circuits. The range of plate load resistances in Table 2 should cover most amateur requirements. Tables 3 and 4 show reactance values for cathode-driven tuned-input circuits for Qo values of 2 and 3 for a range of cathode drive impedances that should cover most requirements. The computer program and printouts are by Don Reaves, KC5JH.

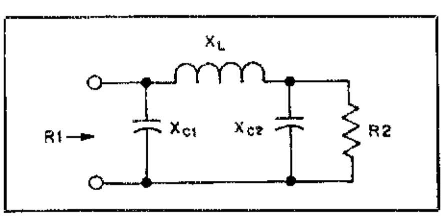

Fig. 3 — Pi-network component designations for use with Procedures 1 and 2. In these procedures, a single set of equations and conditions satisfies both step-up and step-down impedance transformations. Thus, it is unnecessary to specify R1 > R2 or R2 > R1. R1 is the source resistance, and R2 is the load resistance.

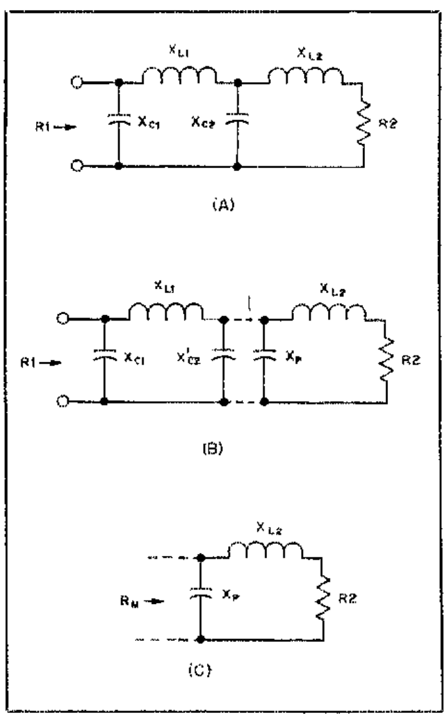

Fig. 4 — The pi-L network, A, is designed as a resonant L network, C, combined with a pi network, as shown at B. X_{C2} is determined through the use of an intermediate value, X'_{C2}, in parallel with Xp.

APPENDIX

For the circuit of Fig. 5A to be equivalent to that of 5B, it is assumed that

$$Z_{in}(A) = Z_{in}(B) \quad \text{(Eq. 22)}$$

$$P_A(A) = P_A(B) = \text{active (real) power} \quad \text{(Eq. 23)}$$

$$P_X(A) = P_X(B) = \text{reactive power} \quad \text{(Eq. 24)}$$

and

$$\frac{Rp}{Xp} = \frac{Xs}{Rs} = Q = \frac{\text{reactive power}}{\text{active power}} \quad \text{(Eq. 25)}$$

$$P_A(A) = I^2Rs = \frac{E^2Rs}{Rs^2 + Xs^2} = P_A(B). = \frac{E^2}{Rp} \quad \text{(Eq. 26)}$$

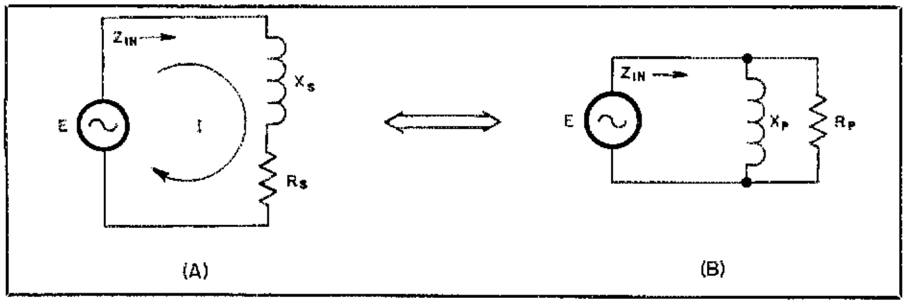

Fig. 5 — A series circuit, A, and its parallel equivalent, B. For the same value of Z_{in} in both drawings, Xs is not equal to Xp, and Rs is not equal to Rp.

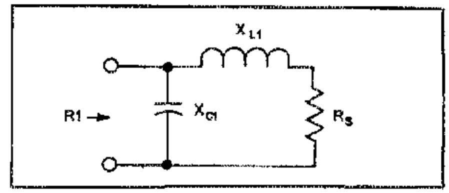

Fig. 7 — The resonant L network, redrawn from Fig. 6A.

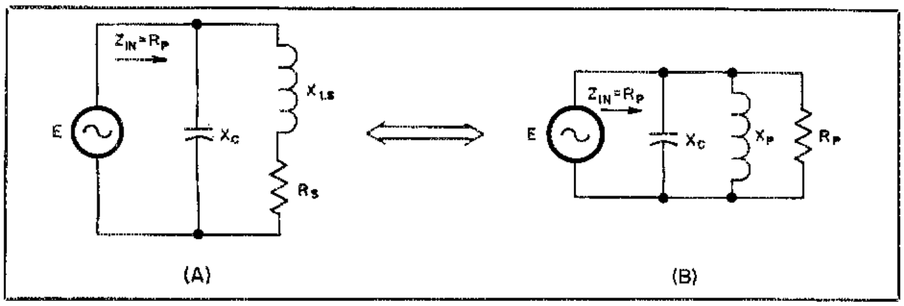

Fig. 6 — The resonant L network is shown at A. At B is the parallel-equivalent circuit.

Therefore,

$$Rp = \frac{Rs^2 + Xs^2}{Rs}$$

$$= Rs\left[1 + \left(\frac{Xs}{Rs}\right)^2\right] = Rs(Q^2 + 1) \quad \text{(Eq. 27)}$$

$$P_X(A) = \frac{E^2Xs}{Rs^2 + Xs} = P_X(B) = \frac{E^2}{Xp} \quad \text{(Eq. 28)}$$

Therefore,

$$Xp = \frac{Rs^2 + Xs^2}{Xs} = \frac{Rs}{Q} + Q \cdot Rs$$

$$= \frac{Rs(Q^2 + 1)}{Q} = \frac{Rp}{Q} \quad \text{(Eq. 29)}$$

From the first terms of Eqs. 27 and 29,

$$Rp \cdot Rs = Xp \cdot Xs \quad \text{(Eq. 30)}$$

From Eqs. 25 and 27,

$$Rs = \frac{Rp}{1 + \left(\frac{Rp}{Xp}\right)^2} = \frac{Rp \cdot Xp^2}{Rp^2 + Xp^2} \quad \text{(Eq. 31)}$$

And from Eq. 25 and 29,

$$Xs = \frac{Rp^2Xp}{Rp^2 + Xp^2} \quad \text{(Eq. 32)}$$

These formulas are for series-to-parallel and parallel-to-series equivalent conversions.

Now see Fig. 6. The series circuit, A, and its parallel equivalent, B, are made parallel resonant by adding a capacitive reactance X_C of a magnitude equal to Xp in shunt, as shown. At the resonant frequency, fo, the circuits appear to the generator as a purely resistive impedance, Z_{in} = Rp. The Q remains the same as before the addition of X_C.

$$Q = Rp/Xp = X_Ls/Rs \quad \text{(Eq. 33)}$$

where X_Ls = Xs in Fig. 5.

The circuit of Fig. 6A is the resonant L net. This circuit is redrawn in Fig. 7 with the circuit elements redesignated for convenience to use in developing the "standard" Q-based pi-network formulas that follow.

$$Q = R1/X_{C1} = X_{L1}/Rs \quad \text{(Eq. 34)}$$

$$R1 = Rs(Q1^2 + 1) \quad \text{(Eq. 35)}$$

where R1 > Rs. Other alternate forms are

$$Q1 = \sqrt{R1/Rs - 1} \quad \text{(Eq. 36)}$$

$$X_{L1} = \sqrt{R1 \cdot Rs - Rs^2} = \frac{Q1 \cdot R1}{Q1^2 + 1} \quad \text{(Eq. 37)}$$

$$X_{C1} = R1\sqrt{Rs/(R1 - Rs)}$$
$$= Rs\,(Q1^2 + 1)/Q1 \quad \text{(Eq. 38)}$$

In the L net, R1 must be greater than Rs. That is, the shunt reactance element is placed in shunt with the larger of the resistances to be matched.

The pi network is divided into two back-to-back L nets, as shown in Fig. 8. Rs is not a physical resistor, but is rather a virtual resistance. It is the transformed value of R2, the load-end resistance value, and is the value of resistance required to afford R1 at the input or source-end L network for the Q1 value selected.

The standard pi-network Q-based formulas follow. Select the Q1 value. Then,

$$X_{C1} = R1/Q1 \quad \text{(Eq. 39)}$$

At the input end in Fig. 8A,

$$Rs = \frac{R1}{Q1^2 + 1} \quad \text{(Eq. 40)}$$

And at the output end,

$$Rs = \frac{R2}{Q2^2 + 1} \quad \text{(Eq. 41)}$$

Table 1

BASIC Program for Designing a Pi Network Using Procedure 1

```
100 REM  PINET4
110 REM Q BASED FORMULAS PER W5FD FOR CALCULATING PI-NETWORK REACTANCE
   VALUES
120 X1=0:X2=0:XL=0:R1=0:R2=0:QO=0:Q1=0:Q2=0
130 INPUT "R1";R1
140 INPUT "R2";R2
150 INPUT"QO";QO
160 IF (R1*R2*QO)<=0 THEN PRINT "Not a pi network": GOTO 340
170 REM  SPECIAL CASE WHERE R1 = R2
180 IF R1=R2 THEN X1=(2*R1)/QO:X2=(2*R2)/QO:Q1=QO/2:XL=(R1*QO)/Q1*Q1+1):GOTO 280
190 IF ABS((QO*QO+1)-(R1/R2))<.01 THEN PRINT "L network": GOTO 340
200 IF ABS((QO*QO+1)-(R2/R1))<.01 THEN PRINT "L network":GOTO 340
210 IF R1/R2>QO*QO+1 THEN PRINT "No solution":GOTO 340
220 R2/R1>QO*QO+1 THEN PRINT "No solution":GOTO 340
230 Q1=(R1*QO - SQR(R1*R2*(QO*QO) - (R1-R2)*(R1-R2)))/(R1-R2)
240 Q2=QO-Q1
250 X1=R1/Q1
260 X2=R2/Q2
270 XL=(R1*QO)/(Q1*Q1+1)
280 PRINT "R1 = ";R1
290 PRINT "R2 = ";R2
300 PRINT "QO = ";QO
310 PRINT "X1 = ";X1
320 PRINT "X2 = ";X2
330 PRINT "XL = ";XL
340 PRINT: RUN
```

Table 2

Pi-Network Calculations for Tube-Type RF Amplifiers

R2 = 50 ohms, Qo = 12

R1	X_{C1}	X_{C2}	X_L
1500	145	31	166
1550	149	31	170
1600	153	32	175
1650	158	33	179
1700	162	33	184
1750	166	34	188
1800	171	34	193
1850	175	35	197
1900	179	36	201
1950	184	36	206
2000	188	37	210
2025	190	37	212
2050	192	37	215
2075	194	38	217
2100	197	38	219
2125	199	38	221
2150	201	39	223
2175	203	39	226
2200	205	39	228
2225	207	39	230
2250	209	40	232
2275	212	40	234
2300	214	40	236
2325	216	41	239
2350	218	41	241
2375	220	41	243
2400	222	42	245
2425	224	42	247
2450	227	42	249
2475	229	42	251
2500	231	43	254
2525	233	43	256
2550	235	43	258
2575	237	44	260
2600	239	44	262
2625	241	44	264
2650	244	45	266
2675	246	45	268
2700	248	45	271
2725	250	46	273
2750	252	46	275
2775	254	46	277
2800	256	47	279
2825	258	47	281
2850	260	47	283
2875	263	48	285
2900	265	48	287
2925	267	48	290
2950	269	49	292
2975	271	49	294
3000	273	49	296
3050	277	50	300
3100	281	51	304
3150	286	51	308
3200	290	52	312
3250	294	53	317
3300	298	54	321
3350	302	54	325
3400	307	55	329
3450	311	56	333
3500	315	57	337
3550	319	57	341
3600	323	58	345
3650	327	59	349
3700	331	60	353
3750	336	61	357
3800	340	62	361
3850	344	62	365
3900	348	63	369
3950	352	64	373
4000	356	65	377

From this,

$$\frac{R1}{Q1^2 + 1} = \frac{R2}{Q2^2 + 1} = \frac{R2}{1 + \dfrac{R2^2}{X_{C2}^2}} \quad \text{(Eq. 42)}$$

Table 3

Pi-Network Calculations for Cathode-Tuned Input Circuits

R1 = 50 ohms, Qo = 2

R2	X_{C1}	X_{C2}	X_L
50	50	50	50
55	53	52	52
60	55	55	55
65	58	57	57
70	60	60	59
75	63	62	61
80	66	65	63
85	68	67	65
90	71	69	67
95	74	72	69
100	77	74	71
105	81	76	72
110	84	78	74
115	88	80	76
120	92	83	77
125	96	85	79
130	100	87	80
135	105	89	81
140	110	91	83
145	115	93	84
150	121	95	85

Table 4

Pi-Network Calculations for Cathode-Tuned Input Circuits

R1 = 50 ohms, Qo = 3

R2	X_{C1}	X_{C2}	X_L
50	33	33	46
55	35	35	48
60	36	38	51
65	37	40	53
70	38	42	55
75	39	44	57
80	40	46	59
85	41	48	61
90	42	49	63
95	43	51	65
100	45	53	66
105	46	55	68
110	47	57	70
115	48	59	72
120	49	61	73
125	50	63	75
130	51	64	77
135	52	66	78
140	53	68	80
145	55	70	81
150	56	71	83

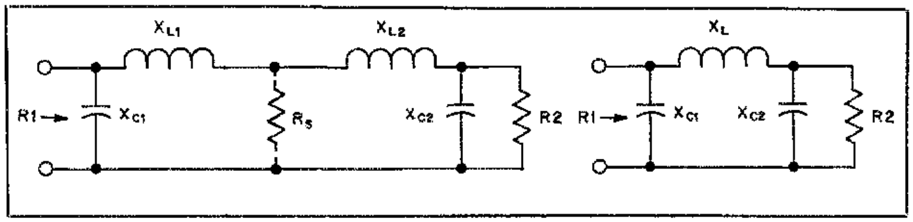

Fig. 8 — The pi network, B, may be considered as two back-to-back L networks, as shown at A. X_L equals the sum of X_{L1} and X_{L2}. Rs is a virtual resistance, not a real resistor.

Solving for X_{C2},

$$X_{C2} = R2\sqrt{\frac{R1/R2}{Q1^2 + 1 - R1/R2}} \quad \text{(Eq. 43)}$$

Further,

$$X_{L1} = Q1 \cdot Rs = \frac{R1 \cdot Q1}{1 + Q1^2} \quad \text{(Eq. 44)}$$

and

$$X_{L2} = \frac{Rs \cdot R2}{X_{C2}} = \frac{R1 \cdot R2}{(Q1^2 + 1)X_{C2}} \quad \text{(Eq. 45)}$$

$$X_L = X_{L1} + X_{L2} = \frac{R1 \cdot Q1 + R1 \cdot R2/X_{C2}}{Q1^2 + 1} \quad \text{(Eq. 46)}$$

Equations 39, 43 and 46 are the old standard Q-based pi-network formulas. They are in slightly different form from those of Fig. 2, but in fact are the same formulas. The formula for X_L may be put in a form involving only Q1, R1 and R2, by substituting the formula for X_{C2} (Eq. 43) into Eq. 46. There is no advantage in doing this, however, because the value of X_{C2} must be found from Eq. 43 and is already available.

These Q-based pi-network formulas give correct results for the network reactance values required for matching R1 to R2 and the selected input-end L-network Q1 value. It is to be noted, however, that this Q1 value is not the pi-network operating Qo value, which is

$$Qo = Q1 + Q2 = R1/X_{C1} + R2/X_{C2} \quad \text{(Eq. 47)}$$

A pi-network solution to match R1 to R2 at the Q value selected for Q1 is possible when $(Q1^2 + 1) > R1/R2$. When $(Q1^2 + 1) = R1/R2$, X_{C2} goes to infinity and an L-net solution results. As Eq. 27 indicates, $Q1^2 + 1 = R1/R2$ is the L-network equation. If $(Q1^2 + 1) < R1/R2$, no solution can be afforded. Note that both R1 and R2 must exceed Rs. (The $Q1^2 + 1 - R1/R2$ term of Eq. 43 takes care of this.)

Now we examine the Qo of pi networks and pi-L networks. In Fig. 9,

$$P_A = \text{active power} = E1^2/R1 = E2^2/R2 \quad \text{(Eq. 48)}$$

$$P_{X1} = \text{reactive power} = E1^2/X_{C1} \quad \text{(Eq. 49)}$$

$$P_{X2} = \text{reactive power} = E2^2/X_{C2} \quad \text{(Eq. 50)}$$

$$Q = \frac{2\pi \text{ max. energy stored}}{\text{energy dissipated per cycle}} = \frac{\text{reactive power}}{\text{active power}} \quad \text{(Eq. 51)}$$

$$Q_\pi = \frac{\dfrac{E1^2}{X_{C1}} + \dfrac{E2^2}{X_{C2}}}{\dfrac{E1^2}{R1}} \quad \text{(Eq. 52)}$$

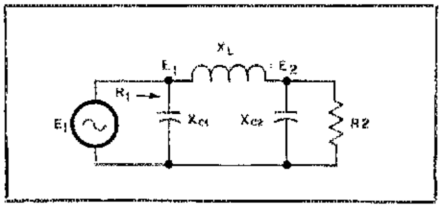

Fig. 9 — The pi network identified for a discussion of the circuit operating Q, Qo.

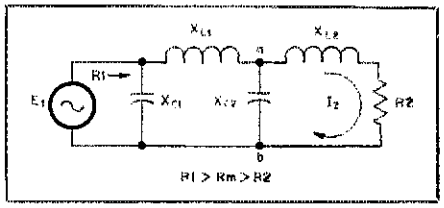

Fig. 10 — The pi-L network identified for a discussion of Qo.

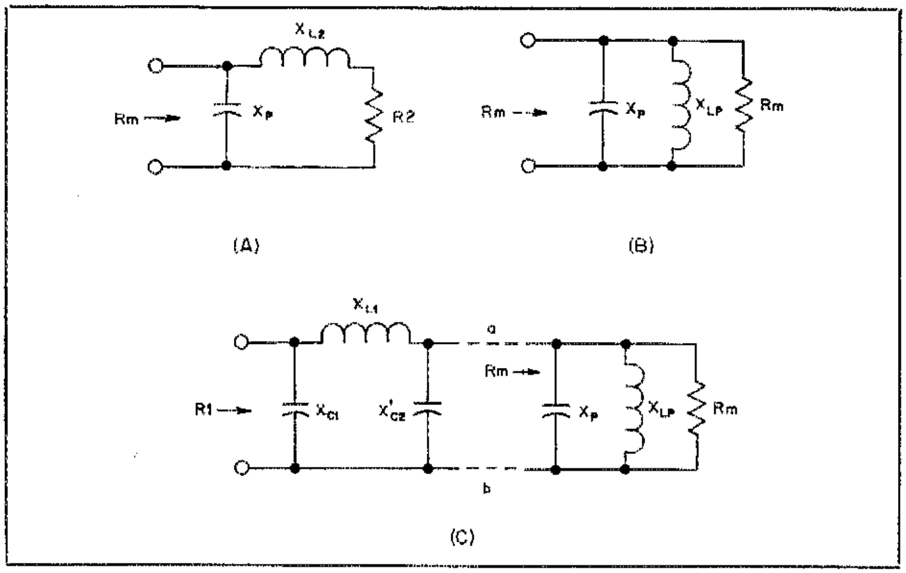

Fig. 11 — Derivation of the pi-L network from an L section, A, and its parallel equivalent, B. The parallel equivalent is added to a pi section, C.

where, from Eq. 48,

$$E2^2 = \frac{R2 \cdot E1^2}{R1} \qquad \text{(Eq. 53)}$$

Therefore,

$$Q_\pi = \frac{R1}{X_{C1}} + \frac{R2}{X_{C2}} = Qo_\pi \text{ (pi-net Qo)} \qquad \text{(Eq. 54)}$$

And for the Qo of the pi-L network, refer to Fig. 10. In that drawing,

$$Z_{AB} = R_{mid} = Rm \qquad \text{(Eq. 55)}$$

where R1 > Rm > R2.

$$Q_{\pi L} = \frac{R1}{X_{C1}} + \frac{Rm}{X_{C2}} = Qo_{\pi L} \text{ (pi-L net Qo)} \qquad \text{(Eq. 56)}$$

The reactive power associated with $I2^2 \cdot X_{L2}$ is included in $E_{AB}{}^2/X_{C2}$.

The Pi-L Network

Refer again to Fig. 4. The circuit Qo value for the pi-L network may be expressed as follows:

$$Qo_{\pi L} = \frac{R1}{X_{C1}} + \frac{Rm}{X_{C2}} \qquad \text{(Eq. 57)}$$

Only the arrangement where R1 > Rm > R2 is considered here, the output-tank-coupling-circuit operation. The Qo of the pi-L is shown as follows. To the series combination of X_{L2} and R2 there is added a capacitive reactance, Xp, to provide a resonant L net with R_{in} = Rm. This is shown in Fig. 4C, as well as in Fig. 11A. In that circuit,

$$Q_L = Rm/Xp = X_{L2}/R2 \qquad \text{(Eq. 58)}$$

The parallel equivalent for this circuit is shown in Fig. 11B. In this circuit,

$$Q = Rm/Xp \qquad \text{(Eq. 59)}$$

Adding the input-end pi network results in the circuit of Fig. 11C. From that diagram,

$$Qo_{\pi L} = \frac{R1}{X_{C1}} + \frac{Rm}{X'_{C2}} + \frac{Rm}{Xp} \qquad \text{(Eq. 60)}$$

$$Qo_{\pi L} = \frac{R1}{X_{C1}} + \frac{Rm}{X_{C2}} \qquad \text{(Eq. 61)}$$

where

$$X_{C2} = \frac{X'_{C2} \cdot Xp}{X'_{C2} + Xp} \qquad \text{(Eq. 62)}$$

$$\frac{Rm}{X'_{C2}} + \frac{Rm}{Xp} = Rm\left(\frac{X'_{C2} + Xp}{X'_{C2} \cdot Xp}\right) = \frac{Rm}{X_{C2}} \qquad \text{(Eq. 63)}$$

Derivation of the Q1 Formula Used in Procedure 1

Refer again to Fig. 8. From the back-to-back L-net pi-network equations,

$$Qo = Q1 + Q2 \qquad \text{(Eq. 64)}$$

$$\frac{R1}{Q1^2 + 1} = \frac{R2}{Q2^2 + 1} \qquad \text{(Eq. 65)}$$

$$Q2^2 = \frac{R2(Q1^2 + 1) - R1}{R1} \qquad \text{(Eq. 66)}$$

$$Q2 = Qo - 1 \qquad \text{(Eq. 67)}$$

$$Q2^2 = (Qo - Q1)^2 \qquad \text{(Eq. 68)}$$

$$\frac{R2(Q1^2 + 1) - R1}{R1} = Qo^2 - 2 \cdot Qo \cdot Q1 + Q1^2 \qquad \text{(Eq. 69)}$$

Solving for Q1 yields

$$Q1 = \frac{Qo - \sqrt{\frac{R2}{R1}\left(Qo^2 + 2 - \frac{R2}{R1}\right) - 1}}{1 - \frac{R2}{R1}} \qquad \text{(Eq. 70)}$$

This formula is rearranged in simpler form and is presented as Eq. 4 in Procedure 1, given earlier. This gives Q1 in terms of the known Qo, R1 and R2, and enables solution equations for the pi-network reactance element values in terms of Qo, R1 and R2.

Derivation of the X_L Formula Used in Procedure 2

Refer now to Fig. 3. Everitt's equations for the low-pass pi network follow.[8]

$$X_{C1} = \frac{R1 \cdot X_L}{R1 + \sqrt{R1 \cdot R2 - X_L{}^2}} \qquad \text{(Eq. 71)}$$

$$X_{C2} = \frac{R2 \cdot X_L}{R2 + \sqrt{R1 \cdot R2 - X_L{}^2}} \qquad \text{(Eq. 72)}$$

$$Q1 = \frac{R1 + \sqrt{R1 \cdot R2 - X_L{}^2}}{X_L} \qquad \text{(Eq. 73)}$$

$$Q2 = \frac{R2 + \sqrt{R1 \cdot R2 - X_L{}^2}}{X_L} \qquad \text{(Eq. 74)}$$

Qo is the circuit Q value, and may be expressed as

$$Qo = Q1 + Q2 \qquad \text{(Eq. 75)}$$

Substituting the Q1 and Q2 expressions in Eq. 75 and solving for X_L yields

$$X_L = \frac{Qo(R1 + R2) + 2\sqrt{R1 \cdot R2(Qo^2 + 4) - (R1 + R2)^2}}{Qo^2 + 4} \qquad \text{(Eq. 76)}$$

This gives X_L in terms of the known Qo, R1 and R2, and enables solutions for X_{C1} and X_{C2} from Everitt's equations or from the equations given in Procedure 2.

The networks discussed in this article are termed linear networks. In addition to other things, this means that they have a simple linear Ohm's law response to an input. It means that these networks work in both directions. That is, a pi, a pi-L or an L network may be designed to transform a load-end R2 value into a value of R1 ohms at the source or input end. If the physical resistor R2 is removed and a physical resistor R1 is placed at the R1 end terminals, then this value of R1 will have been transformed into a value of R2 ohms at the R2 end terminals.

It is to be noted that there is no restriction placed on the ratio of R1 to R2 in the new formulas given in this article. This is unlike the way in which the old standard Q-based pi-network formulas were normally stated, in which R1 had to exceed R2. The new formulas work equally well, whether R1 is greater than or less than R2. The only restriction is in the requirement that $Qo^2 + 1$ must exceed the higher value of R1/R2 or R2/R1. The requirement that R1 had to exceed R2, as the old standard formulas were normally stated, was never correct and, in fact, the old standard formulas work equally well for R1 > R2 or R1 < R2, as an application to an example pi network solution will indicate. This is in consequence of the fact that the old standard Q-based formulas did and do provide an exact "mathematical model" of the physical pi network insofar as computing the reactive network element values for the R1-to-R2 transformation. The only shortcoming of the old formulas is that the computation was based on a partial circuit Q value, and this partial value was taken as the circuit operating Q value. Even so, once the network was solved, the correct operating Qo value was readily available by adding the partial circuit Q value that was not accounted for. The new formulas allow the network solution based on the correct circuit Qo value selected at the outset.

Notes

[1]E. W. Pappenfus and K. L. Klippel, "Pi Network Tank Circuits, *CQ*, Sept. 1950, p. 27.

[2]F. Davis, "Matching Network Designs with Computer Solutions," *Application Note 267*, Motorola, Inc., p. 6.

[3]H. L. Gibson, "An Improved Design Method for Pi and L Pi Network Couplers," *Radio Communication*, June 1969, p. 390.

[4]*Radio Data Reference Book* (3rd edition). London: RSGB, 1972.

[5]E. W. Whyman, "Pi-Network Design and Analysis," *Ham Radio*, Sept. 1977, p. 30.

[6]K. Kajii, "Design of Pi-Type Circuits," *CQ Ham Radio* (Japan), June 1974, p. 264.

[7]L. Everitt, *Communication Engineering*, 1st edition (New York and London: McGraw-Hill Book Company, Inc., 1932).

[8]See note 7.

New Books

☐ *70 Years of Radio Tubes and Valves* by John W. Stokes. Published by the Vestal Press Ltd., Vestal, NY 13850. First edition, 1982. Hardbound, 8-3/4 × 11 inches, 247 pp., $21.95.

☐ Amidst the furor of home computers, large-scale integration and satellite TV, there remains a small, but highly devoted contingent of vacuum-tube enthusiasts. And why not? Admittedly, present-day use of tubes is largely restricted to high-power (usually transmitting) applications, but not a single modern-day radio circuit has developed without some footing in tube-based principles. For this reason alone, we owe homage to the vacuum tube with books that preserve the history of this noble device. *70 Years of Radio Tubes and Valves* is such a book.

Even today's "silicon generation" should find the story of vacuum tubes' golden age extremely interesting — or perhaps fascinating is a better word. John Stokes does much to promote this fascination in *70 Years*, which gives the book broad-based appeal.

The title may seem a bit misleading, for what, if *any*, difference is there between a valve and a tube? Technically, there is none. But Stokes, who is a New Zealander, writes as an impartial historian of the parallel but distinctly separate developments in the American and European tube industries. With American developments, he refers to the vacuum tube, while with European counterparts he refers to the valve, after Fleming's early name for the device.

Stokes has ordered his book according to the chronological development of the tube. He begins by documenting the early experiments of Edison, Fleming and DeForest, while carefully noting the ambiguity of this developmental period — primarily surrounding each man's claim to be *the* original inventor. As the book progresses, entire chapters are devoted to specific aspects of tube development. For example, there are chapters entitled, "The Grid," "Metal Envelopes," "Penta-Hodas" and "Miniaturization." Each chapter is extremely well-researched and so packed with information that I occasionally found it necessary to stop reading and digest a bit. My only criticism (and a minor one at that) stems from Stokes' tendency to cram perhaps a wee bit too much information into each chapter.

It should be noted that readers specifically interested in the developments of companies such as Telefunken and Phillips will not be satisfied by this book. Stokes has intentionally limited his research to the USA and England, citing language difficulties as the reason. These companies are mentioned (one minor chapter is on Phillips), but only when their achievements affected the course of tube development on a worldwide basis.

Some mention should be made of the excellent selection of photos and graphics in *70 Years*. There is an abundance of original tube advertisements, which alone often tell the state of the art at that time. It's interesting to see ads that directly brand competitors' products unauthorized under patent law. Also, there was a great deal of emphasis placed on "magic breakthroughs" in early tube technology that could easily cure *all* receiver problems — much the same as "snake oil" for your health! Photos are equally interesting, and practically every tube type that is mentioned finds its way into a photograph or excellent illustration.

As the subtitle, "...A Guide for Electronic Engineers Historians and Collectors," claims this book should prove to be a valuable reference work both now and in the future. You don't fit into any of the categories? Don't despair, for *70 Years* will prove to be interesting to *all* who have at least a passing fancy on the subject. — *Dennis J. Lusis, W1LJ*

Strays

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